



CHAPTER 10

Determining
How Costs
Behave

COST FUNCTIONS

A cost function is a mathematical representation of how a cost changes with changes in the level of an activity relating to that cost.

COST TERMINOLOGY

Variable costs—costs that change in total in relation to some chosen activity or output

Fixed costs—costs that do not change in total in relation to some chosen activity or output

Mixed costs—costs that have both fixed and variable components; also called semivariable costs

COST FUNCTION ASSUMPTIONS

1. Variations in the level of a single activity (the cost driver) explain the variations in the related total costs.
2. Cost behavior is approximated by a linear cost function within the relevant range.
 - Graphically, the total cost versus the level of a single activity related to that cost is a straight line within the relevant range.

BRIDGING ACCOUNTING AND STATISTICAL TERMINOLOGY

Accounting	Statistics
Variable Cost	Slope
Fixed Cost	Intercept
Mixed Cost	Linear Cost Function

LINEAR COST FUNCTION

$$y = a + bX$$

The dependent variable:
the cost that is
being predicted

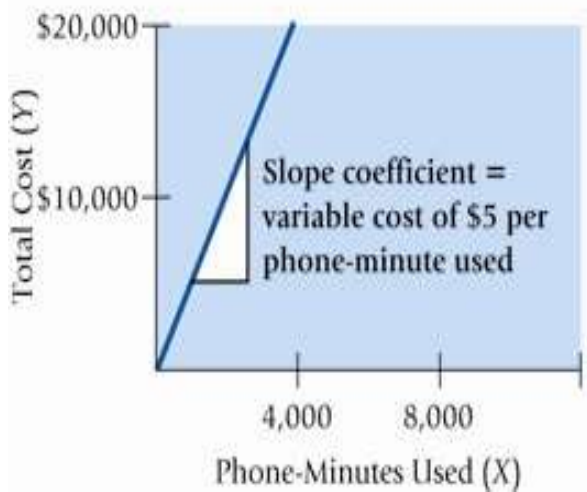
The independent
variable:
the cost driver

The intercept:
fixed costs

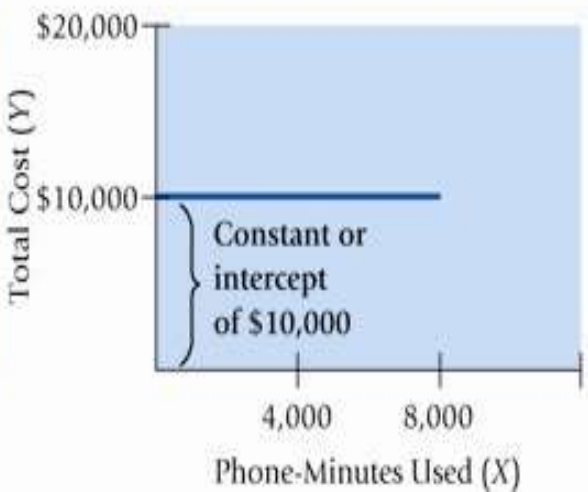
The slope of
the line:
variable cost
per unit

LINEAR COST FUNCTIONS ILLUSTRATED

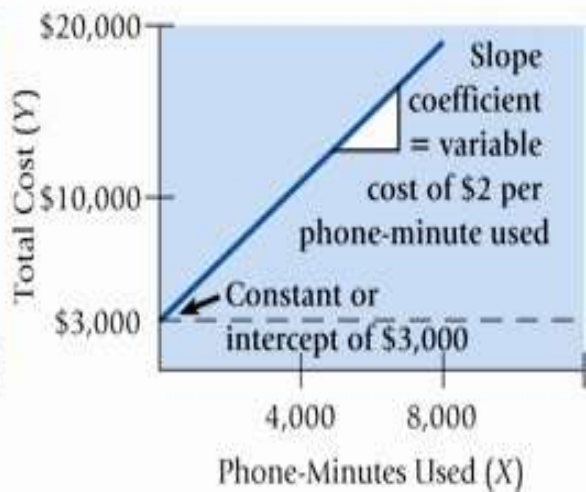
PANEL A:
Variable Cost



PANEL B:
Fixed Cost



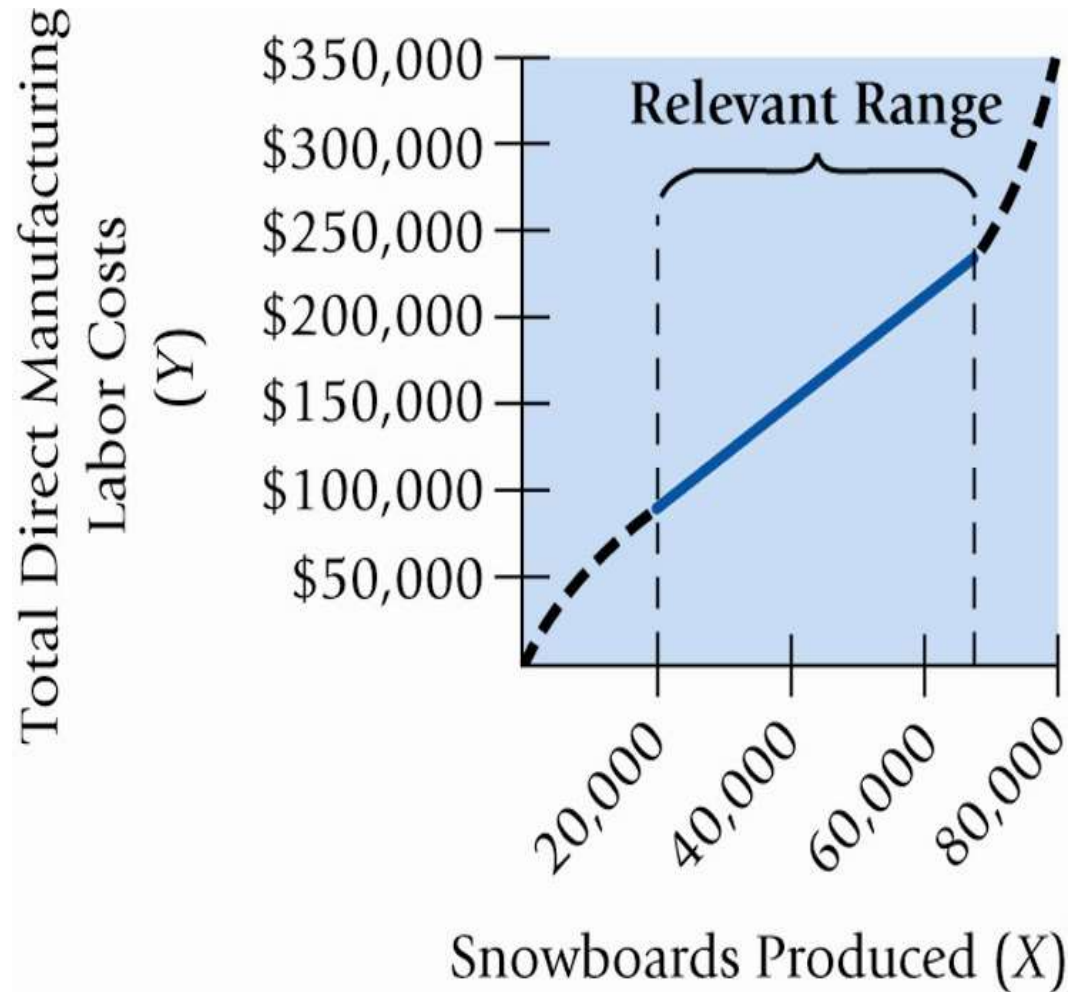
PANEL C:
Mixed Cost



CRITERIA FOR CLASSIFYING VARIABLE AND FIXED COMPONENTS OF A COST

1. Choice of cost object—different objects may result in different classification of the same cost
2. Time horizon—the longer the period, the more likely the cost will be variable
3. Relevant range—behavior is predictable only within this band of activity

THE RELEVANT RANGE ILLUSTRATED



CAUSE AND EFFECT AS IT RELATES TO COST DRIVERS

The most important issue in estimating a cost function is determining whether a cause-and-effect relationship exists between the level of an activity and the costs related to that level of activity.

CAUSE AND EFFECT AS IT RELATES TO COST DRIVERS

A cause-and-effect relationship might arise as a result of:

- A physical relationship between the level of activity and costs
- A contractual agreement
- Knowledge of operations

Note: A high correlation (connection) between activities and costs does not necessarily mean causality.

COST ESTIMATION METHODS

1. Industrial engineering method
2. Conference method
3. Account analysis method
4. Quantitative analysis methods
 1. High-low method
 2. Regression analysis

INDUSTRIAL ENGINEERING METHOD

Estimates cost functions by analyzing the relationship between inputs and outputs in physical terms

Includes time-and-motion studies

Very thorough and detailed, but also costly and time-consuming

Also called the work-measurement method

CONFERENCE METHOD

Estimates cost functions on the basis of analysis and opinions about costs and their drivers gathered from various departments of a company

Pools expert knowledge

Reliance on opinions still makes this method subjective

ACCOUNT ANALYSIS METHOD

Estimates cost functions by classifying various cost accounts as variable, fixed, or mixed with respect to the identified level of activity

Is reasonably accurate, cost-effective, and easy to use, but is subjective

QUALITATIVE ANALYSIS

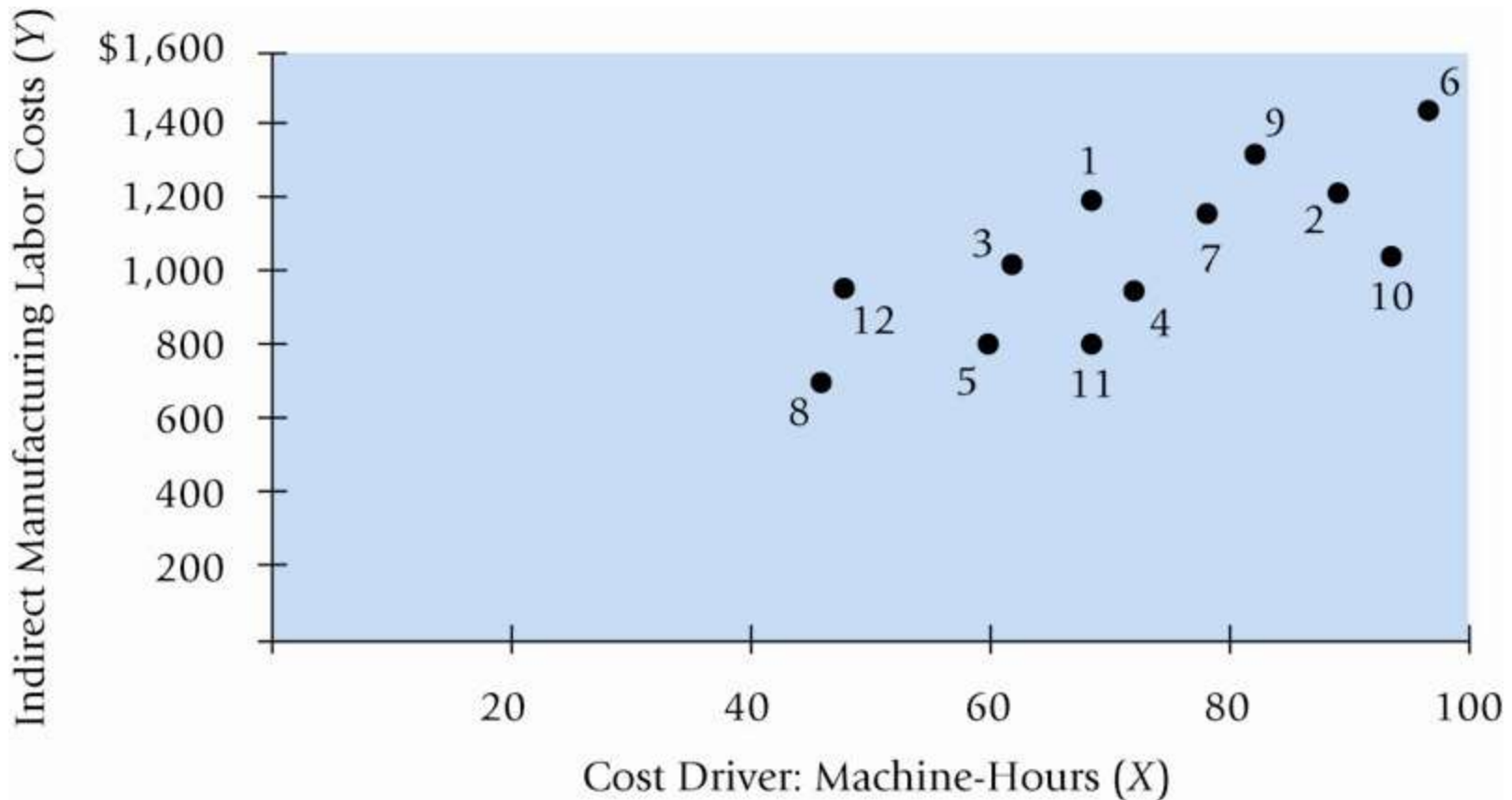
Uses a formal mathematical method to fit cost functions to past data observations

Advantage: results are objective

STEPS IN ESTIMATING A COST FUNCTION USING QUANTITATIVE ANALYSIS

1. Choose the dependent variable (the cost to be predicted).
2. Identify the independent variable or cost driver.
3. Collect data on the dependent variable and the cost driver.
4. Plot the data.
5. Estimate the cost function using the high-low method or regression analysis.
6. Evaluate the cost driver of the estimated cost function.

SAMPLE COST—ACTIVITY PLOT

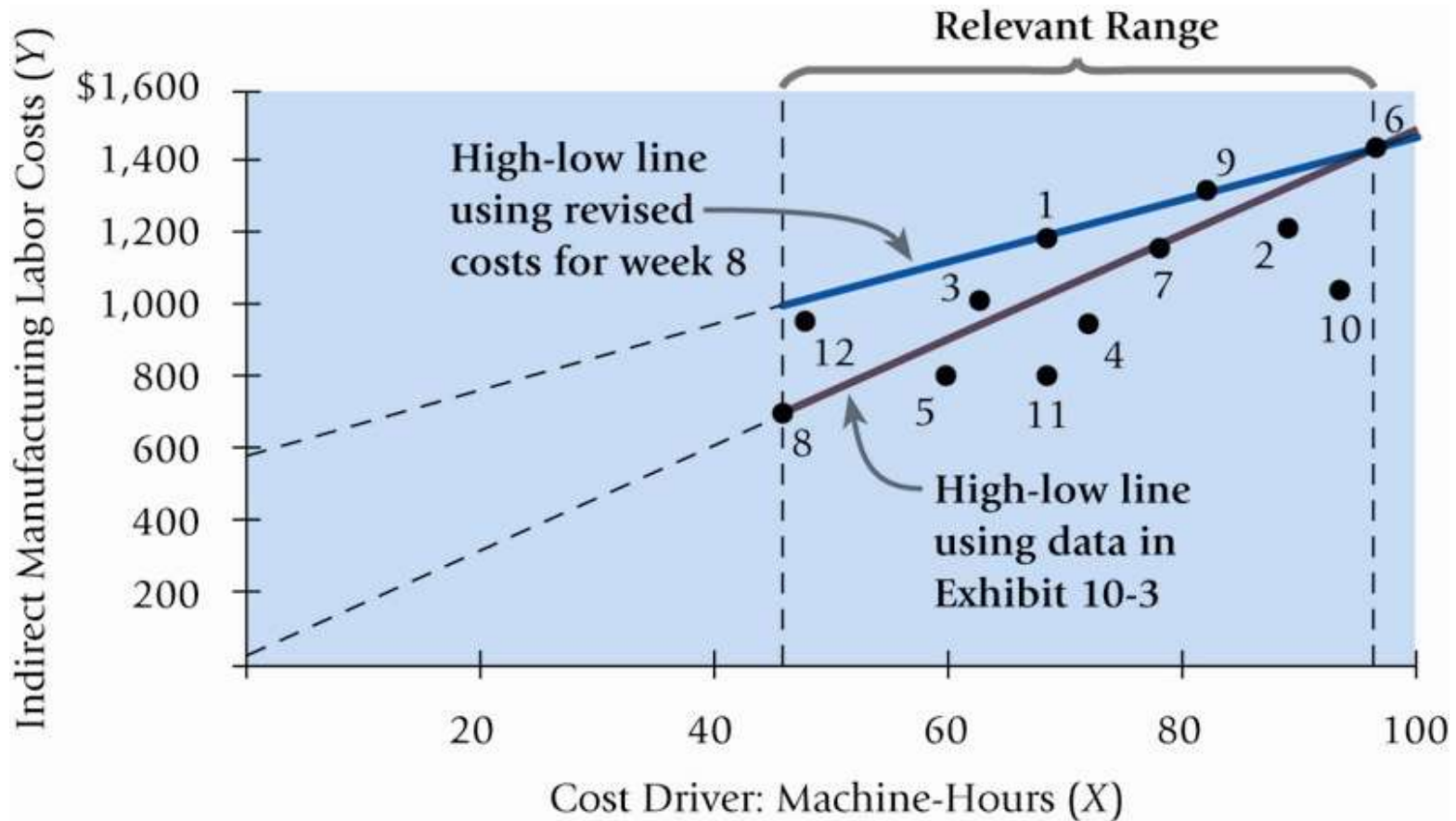


HIGH-LOW METHOD

Simplest method of quantitative analysis

Uses only the highest and lowest observed values

HIGH-LOW METHOD PLOT



STEPS IN THE HIGH-LOW METHOD

1. Calculate variable cost per unit of activity.

	Variable		}	Cost associated with		Cost associated with	}
	Cost per	=		highest activity level	-	lowest activity level	
	Unit of Activity			Highest activity level	-	Lowest activity level	

STEPS IN THE HIGH-LOW METHOD

2. Calculate total fixed costs.

	Total Cost from either the highest or lowest activity level
	- (Variable Cost per unit of activity X Activity associated with above total cost)
	Fixed Costs

3. Summarize by writing a linear equation.

$$Y = \text{Fixed Costs} + (\text{Variable cost per unit of Activity} * \text{Activity})$$

$$Y = FC + (VC_u * X)$$

REGRESSION ANALYSIS

Regression analysis is a statistical method that measures the average amount of change in the dependent variable associated with a unit change in one or more independent variables.

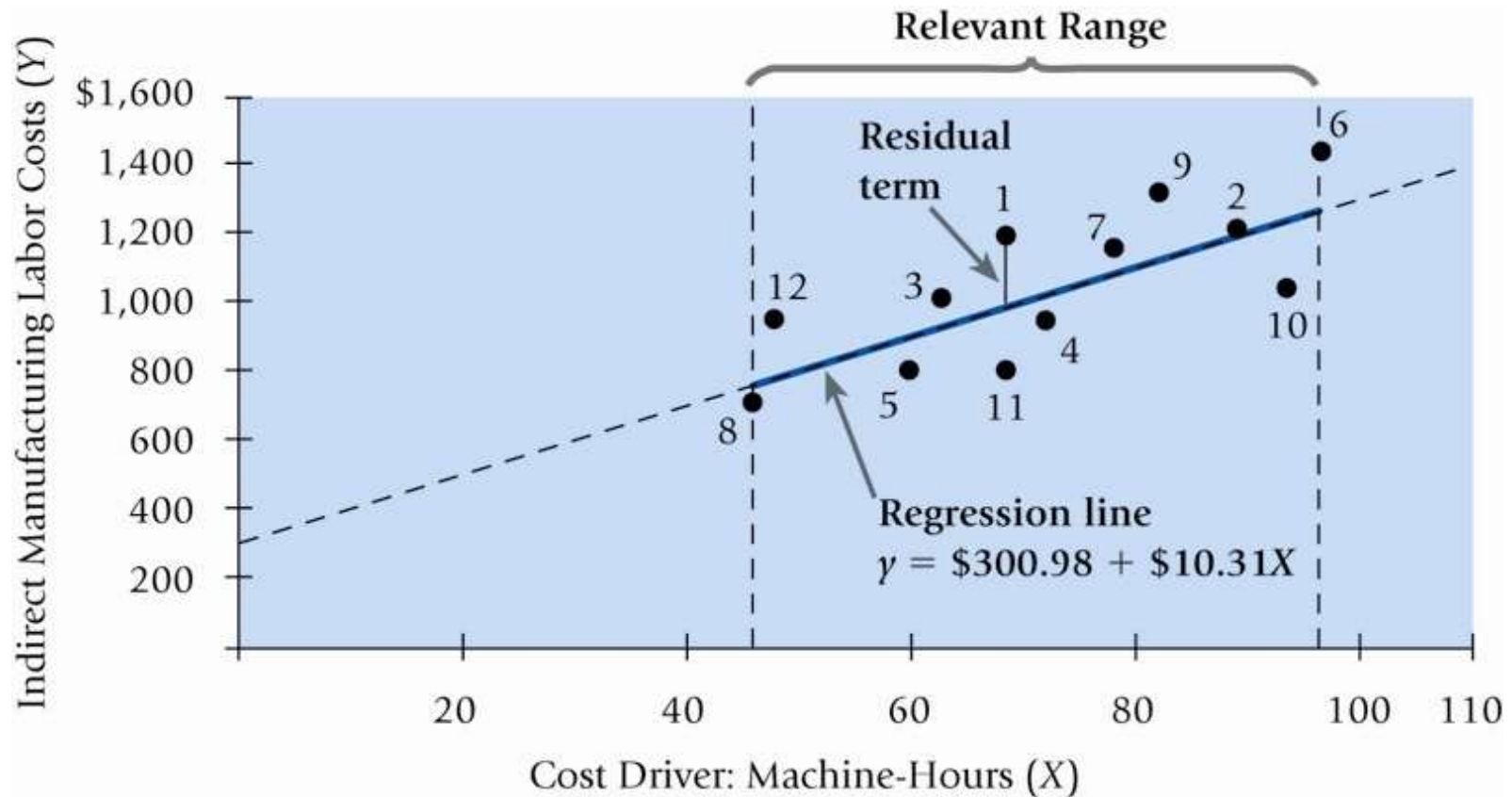
Is more accurate than the high-low method because the regression equation estimates costs using information from *all* observations; the high-low method uses only *two* observations.

TYPES OF REGRESSION

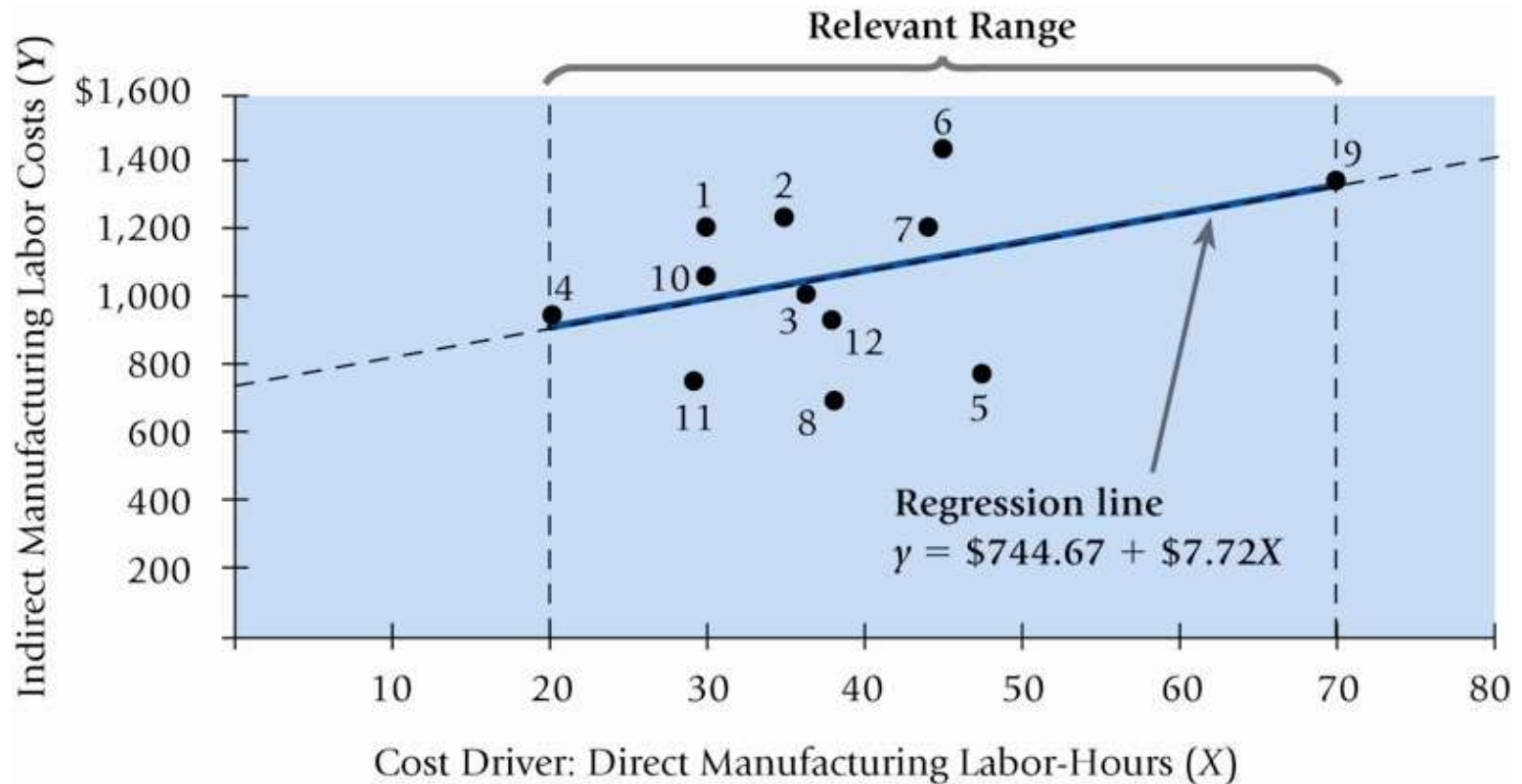
Simple—estimates the relationship between the dependent variable and *one* independent variable

Multiple—estimates the relationship between the dependent variable and *two or more* independent variables

SAMPLE REGRESSION MODEL PLOT



ALTERNATIVE REGRESSION MODEL PLOT



TERMINOLOGY

Goodness of fit—indicates the strength of the relationship between the cost driver and costs

Residual term—measures the distance between actual cost and estimated cost for each observation

CRITERIA FOR EVALUATING ALTERNATIVE COST DRIVERS

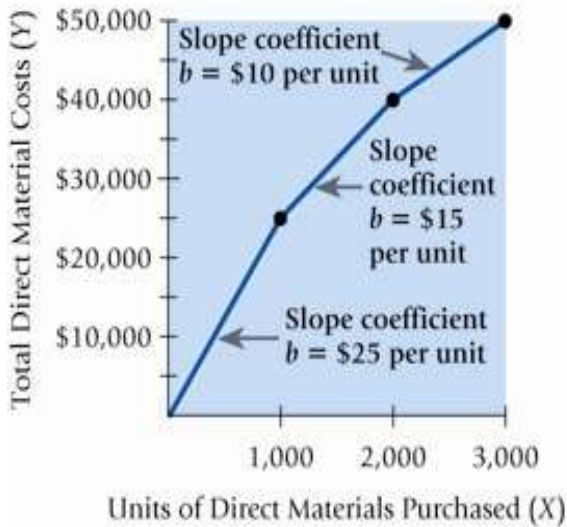
1. Economic plausibility
2. Goodness of fit
3. Significance of the independent variable

NONLINEAR COST FUNCTIONS

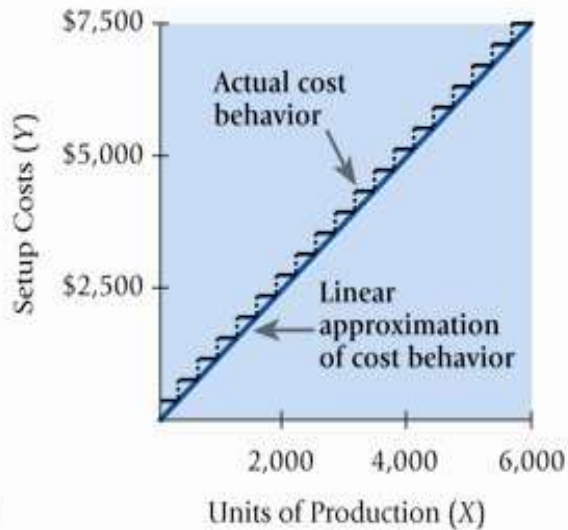
1. Economies of scale
2. Quantity discounts
3. Step cost functions—resources increase in “lot-sizes”, not individual units
4. Learning curves—labor hours consumed decrease as workers learn their jobs and become better at them
5. Experience curve —broader application of learning curve that includes downstream activities including marketing and distribution

NONLINEAR COST FUNCTIONS ILLUSTRATED

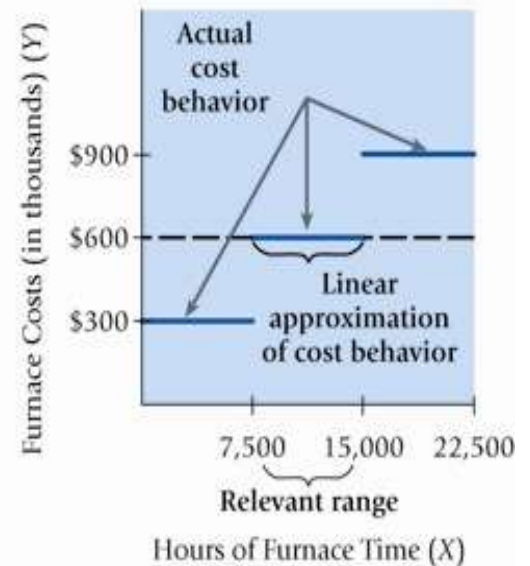
PANEL A:
Effects of Quantity Discounts on Slope Coefficient of Direct Material Cost Function



PANEL B:
Step Variable-Cost Function



PANEL C:
Step Fixed-Cost Function



TYPES OF LEARNING CURVES

Cumulative average-time learning model—*cumulative average time per unit* declines by a constant percentage each time the cumulative quantity of units produced doubles

Incremental unit-time learning model—*incremental time needed to produce the last unit* declines by a constant percentage each time the cumulative quantity of units produced doubles

SAMPLE CUMULATIVE AVERAGE-TIME MODEL

	A	B	C	D	E	F	G	H	I
1	Cumulative Average-Time Learning Model for Rayburn Corporation								
2									
3		80% Learning Curve							
4									
5	Cumulative	Cumulative		Cumulative	Individual Unit				
6	Number	Average Time		Total Time:	Time for Xth				
7	of Units (X)	per Unit (y)*: Labor Hours		Labor-Hours	Unit: Labor Hours				
8									
9			D = Col A x Col B						
10									
11	1	100.00		100.00	100.00				
12	2	80.00	=(100x0.8)	160.00	60.00				
13	3	70.21		210.63	50.63				
14	4	64.00	=(80x0.8)	256.00	45.37				
15	5	59.56		297.82	41.82				
16	6	56.17		337.01	39.19				
17	7	53.45		374.14	37.13				
18	8	51.20	=(64x0.8)	409.60	35.46				
19	9	49.29		443.65	34.05				
20	10	47.65		476.51	32.86				
21	11	46.21		508.32	31.81				
22	12	44.93		539.22	30.89				
23	13	43.79		569.29	30.07				
24	14	42.76		598.63	29.34				
25	15	41.82		627.30	28.67				
26	16	40.96	=(51.2x0.8)	655.36	28.06				
27									

E13 = D13 - D12
= 210.63 - 160.00

*The mathematical relationship underlying the cumulative average-time learning model is:

$$y = aX^b$$

where y = Cumulative average time (labor-hours) per unit
 X = Cumulative number of units produced
 a = Time (labor-hours) required to produce the first unit
 b = Factor used to calculate cumulative average time to produce units

The value of b is calculated as

$$\ln(\text{learning-curve \% in decimal form}) / \ln 2$$

For an 80% learning curve, $b = \ln 0.8 / \ln 2 = -0.2231 / 0.6931 = -0.3219$
 when $X = 3$, $a = 100$, $b = -0.3219$,

$$y = 100 \times 3^{-0.3219} = 70.21 \text{ labor hours}$$

Numbers in table may not be exact because of rounding.

SAMPLE INCREMENTAL UNIT-TIME MODEL

	A	B	C	D	E	F	G	H	I	
1	Incremental Unit-Time Learning Model for Rayburn Corporation									
2										
3	80% Learning Curve									
4										
5	Cumulative	Individual Unit Time		Cumulative	Cumulative					
6	Number	for Xth Unit (y)*:		Total Time:	Average Time					
7	of Units (X)	Labor Hours		Labor-Hours	per Unit:					
8					Labor-Hours					
9										
10					E = Col D ÷ Col A					
11										
12	1	100.00		100.00	100.00					
13	2	80.00	=(100x0.8)	180.00	90.00					
14	3	70.21		250.21	83.40					
15	4	64.00	=(80x0.8)	314.21	78.55					
16	5	59.56		373.77	74.75					
17	6	56.17		429.94	71.66					
18	7	53.45		483.39	69.06					
19	8	51.20	=(64x0.8)	534.59	66.82					
20	9	49.29		583.89	64.88					
21	10	47.65		631.54	63.15					
22	11	46.21		677.75	61.61					
23	12	44.93		722.68	60.22					
24	13	43.79		766.47	58.96					
25	14	42.76		809.23	57.80					
26	15	41.82		851.05	56.74					
27	16	40.96	=(51.2x0.8)	892.01	55.75					
28										

$$D14 = D13 + B14$$

$$= 180.00 + 70.21$$

*The mathematical relationship underlying the incremental unit-time learning model is:

$$y = aX^b$$

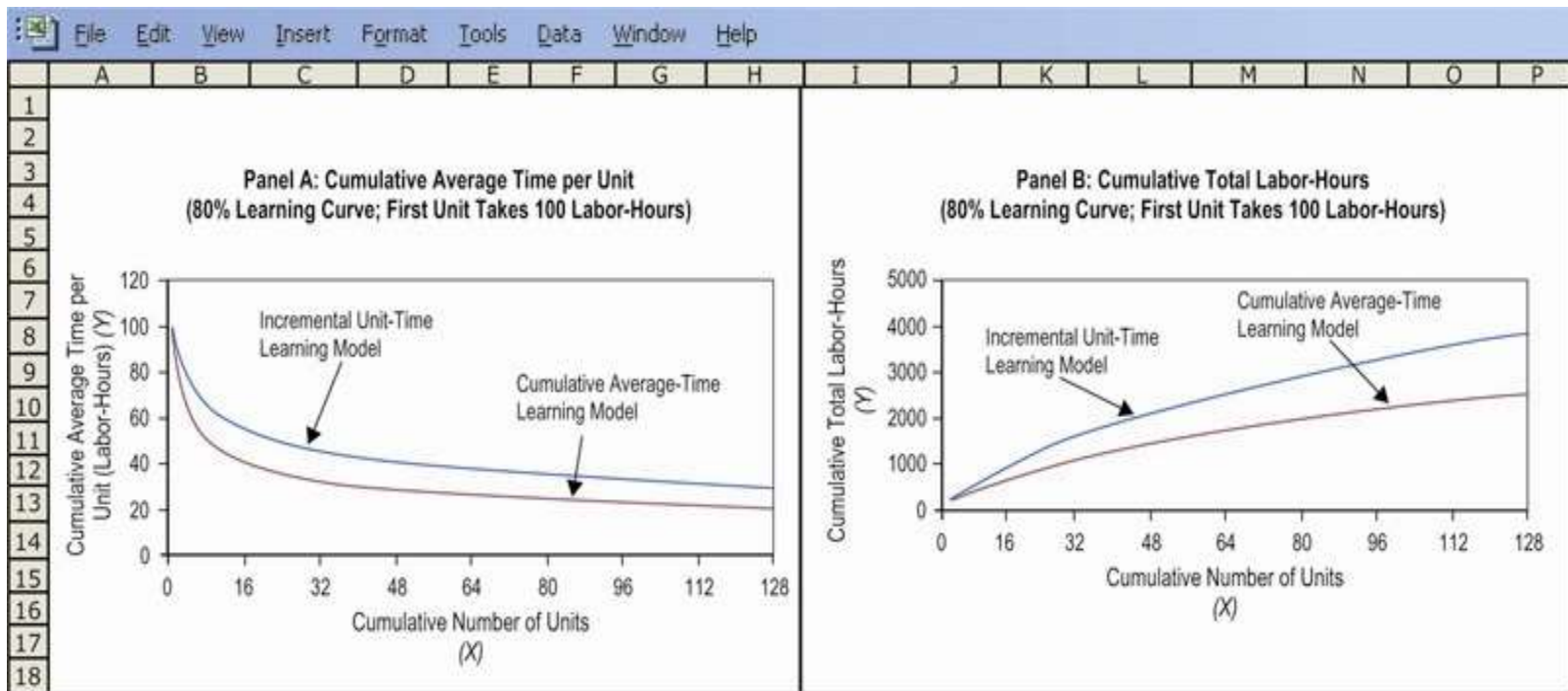
where:

- y = Time (labor-hours) taken to produce the last single unit
- X = Cumulative number of units produced
- a = Time (labor-hours) required to produce the first unit
- b = Factor used to calculate incremental unit time to produce units

$$b = \frac{\ln(\text{learning-curve \% in decimal form})}{\ln 2}$$

For an 80% learning curve, $b = \ln 0.8 \div \ln 2 = -0.2231 \div 0.6931 = -0.3219$
 Where $X = 3$, $a = 100$, $b = -0.3219$,
 $y = 100 \times 3^{-0.3219} = 70.21$ labor hours.
 The cumulative total time when $X = 3$ is $100+80+70.21=250.21$ labor-hours.
 Numbers in the table may not be exact because of rounding.

TIME LEARNING MODEL COMPARATIVE PLOTS



PREDICTING COSTS USING ALTERNATIVE TIME LEARNING MODELS

	A	B	C	D	E	F
1		Cumulative				
2	Cumulative	Average Time	Cumulative	Cumulative Costs		Additions to
3	Number of	per Unit:	Total Time:	at \$50 per		Cumulative
4	Units	Labor-Hours^a	Labor-Hours^a	Labor-Hour		Costs
5	1	100.00	100.00	\$ 5,000	(100.00 x \$50)	\$ 5,000
6	2	80.00	160.00	8,000	(160.00 x \$50)	3,000
7	4	64.00	256.00	12,800	(256.00 x \$50)	4,800
8	8	51.20	409.60	20,480	(409.60 x \$50)	7,680
9	16	40.96	655.36	32,768	(655.36 x \$50)	12,288
10						
11	^a Based on the cumulative average-time learning model. See Exhibit 10-10 for the computations					
12	of these amounts.					

THE IDEAL DATABASE

1. The database should contain numerous reliably measured observations of the cost driver and the costs.
2. In relation to the cost driver, the database should consider many values spanning a wide range.

DATA PROBLEMS

The time period for measuring the dependent variable does not match the period for measuring the cost driver.

Fixed costs are allocated as if they are variable.

Data are either not available for all observations or are not uniformly reliable.

DATA PROBLEMS

Extreme values of observations occur from errors in recording costs.

There is no homogeneous relationship between the cost driver and the individual cost items in the dependent variable-cost pool. A homogeneous relationship exists when each activity whose costs are included in the dependent variable has the same cost driver.

DATA PROBLEMS

The relationship between the cost driver and the cost is not stationary.
Inflation has affected costs, the driver, or both.



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